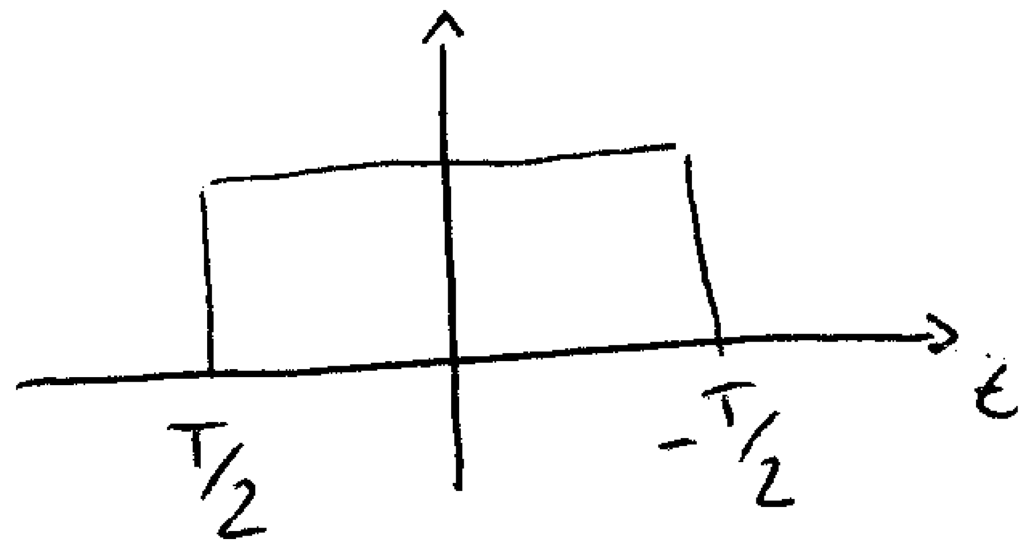
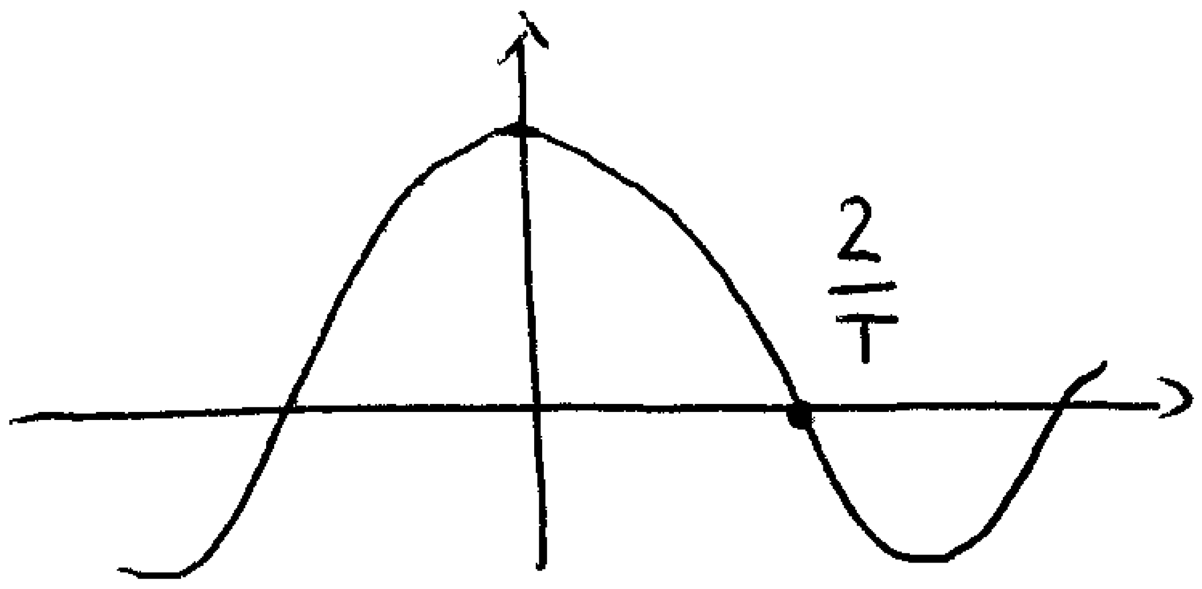
Ex 1

$h(t) = P_{\frac{T}{2}}(t)$ dove



$H(f) =$



1) $f_0 = \frac{2}{T}$ $\quad T = \frac{2}{f_0} = \frac{2}{3 \cdot 10^3} = 6{,}67 \cdot 10^{-4}\,s \quad 0{,}667\ ms$

2) $h(t) = e^{-\frac{1}{2}t} u(t)$

$\hookrightarrow H(f) = \frac{1}{a + j2\pi f}$ con $a = \frac{1}{2}$

$|H(f)|^2 = \frac{1}{a + j2\pi f} \cdot \frac{1}{a - j2\pi f} = \frac{1}{a^2 + 4\pi^2 f^2}$

$|H(f)|^2 = \frac{1}{\frac{1}{4} + 4\pi^2 f^2} = \frac{4}{1 + 16\pi^2 f^2}$

$|H(0)|^2 = 4$

$|H(f)|^2 = 2 \quad \frac{4}{1+16\pi^2 f^2} = 2 \quad 4 = 2 + 32\pi^2 f^2 = 2 = 32\pi^2 f^2$

$f = \sqrt{\frac{1}{16\pi^2}} = \frac{1}{4\pi} \quad \boxed{f = \frac{1}{4\pi}}$

Ex 2

$$P_1(e) = \binom{5}{1} p(1-p)^4 = \frac{5!}{4!} p(1-p)^4 = 5p(1-p)^4 = 0.005$$

$$P_2(e) = \binom{5}{2} p^2(1-p)^3 = \frac{5!}{2!\,3!} p^2(1-p)^3 = \frac{5\cdot 4}{2} p^2(1-p)^3 = 10p^2(1-p)^3 =$$
$$= 10^{-5}$$

$$P_3(e) = \binom{5}{3} p^3(1-p)^2 = 10p^3(1-p)^2 = 10^{-8}$$

$$P_{\leq 3}(e) = P_1(e) + P_2(e) + P_3(e) = \binom{5}{1} p(1-p)^4 + \binom{5}{2} p^2(1-p)^3 + \binom{5}{3} p^3(1-p)^2$$

$$\simeq \binom{5}{1} p(1-p)^4$$

$$p \simeq 5 \cdot 10^{-3}$$

$$\binom{5}{0} p^0 (1-p)^5 \simeq 0{,}9999$$

$$x(t) = A + \cos(2\pi f_0 t + \varphi) \qquad \phi = [-\pi; \pi] \qquad A = [1, 5]$$

fisso A_1 e ϕ_1 per la media temporale

$$\langle x_1(t) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left[A_1 + \cos(2\pi f_0 t + \varphi_1) \right] dt = \frac{A_1}{T} \int_{-T/2}^{T/2} \left[1 + \cancelto{0}{\frac{\cos(2\pi f_0 t + \varphi_1)}{A_1}} \right] dt$$

$$\langle x_1(t) \rangle = A_1$$

$$E\{x(t)\} = E_x\{x(t)\} = E_{A,\phi}\{A + \cos(2\pi f_0 t + \phi)\}$$

$$= E\{A\} + E\{\cancelto{0}{\cos(2\pi f_0 t + \phi)}\}$$

$$\int_{-\pi}^{\pi} \frac{1}{2\pi} \cos(2\pi f_0 t + \phi)\, d\phi$$

$$= E\{A\} = \frac{1}{4} \int_1^5 x\, dx = \frac{1}{4} \left. \frac{x^2}{2} \right|_1^5 = \frac{1}{4}\left(\frac{25}{2} - \frac{1}{2} \right) = \frac{24}{2} \cdot \frac{1}{4} = 3$$

$$R_x(\tau) = E_x\{x(t) \cdot x(t-\tau)\} = E_{A,\phi}\{A^2 + \cos(2\pi f_0 t)\cos(2\pi f_0 (t-\tau) + \phi)$$
$$+ A\cos(2\pi (t-\tau) f_0 + \phi) + A\cos(2\pi f_0 t + \phi)\}$$

$$= E_A\{A^2\} + E_\phi\{\cos(2\pi f_0 t + \phi)\cos(2\pi f_0 (t-\tau) + \phi)\} + E_{A,\phi}\{\cancelto{0}{A\cos(2\pi f_0 (t-\tau) + \phi)}\}$$

$$+ E_{A,\phi}\{\cancelto{0}{A\cos(2\pi f_0 t + \phi)}\}$$

$$= E_A\{A^2\} + E_\phi\left\{\frac{1}{2}\cos(2\pi f_0 \tau) + \underbrace{\cancel{\frac{1}{2}\cos\left(2\pi(2t-\tau)f_0 + 2\phi\right)}}_{0}\right\}$$

$$= E_A\{A^2\} + E_\phi\left\{\frac{1}{2}\cos(2\pi f_0 \tau)\right\}$$

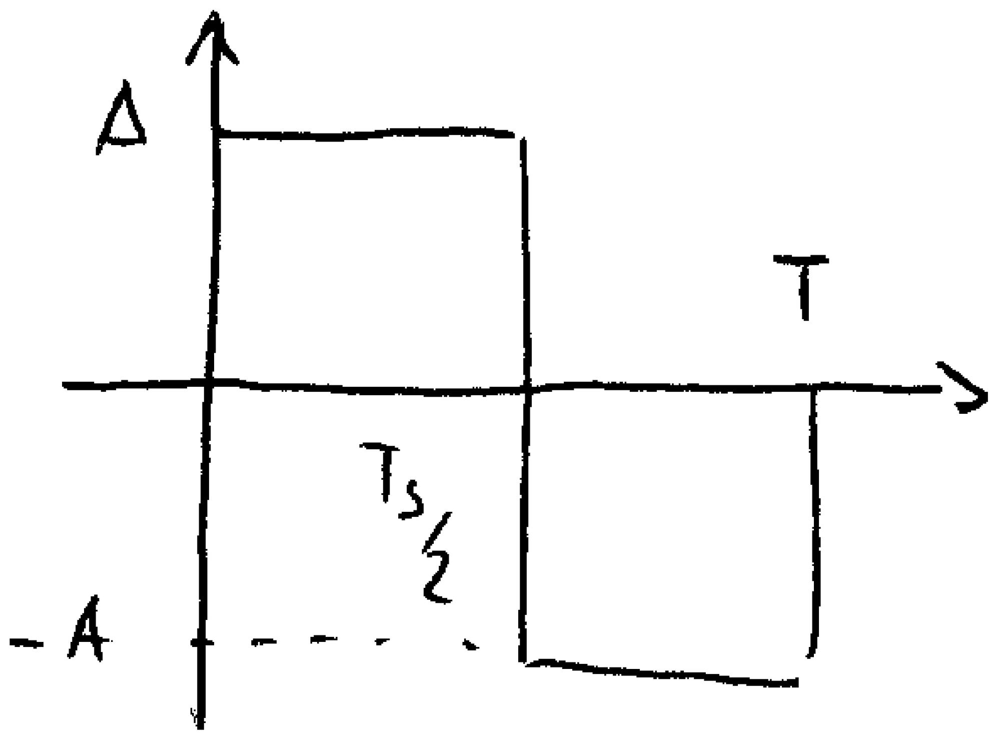
$$\downarrow$$

$$\int_1^5 \frac{1}{4} A^2 \, da = \frac{1}{4}\frac{A^3}{3}\Bigg|_1^5 = \frac{1}{4}\left(\frac{125}{3} - \frac{1}{3}\right) = \frac{124}{3}\cdot\frac{1}{4}$$

$$R_x(\tau) = \frac{31}{3} + \frac{1}{2}\cos(2\pi f_0 \tau)$$

$$\downarrow$$

$$S_x(f) = \frac{31}{3}\delta(f) + \frac{1}{4}\delta(f-f_0) + \frac{1}{4}\delta(f+f_0)$$



$$E_s = A^2 T = \frac{4}{9} T$$

$$\frac{1}{3} = \frac{4}{9} T \qquad \boxed{T = \frac{3}{4}}$$

$$S(t) = A P_{T/2}\left(t - \tfrac{T}{4}\right) - A P_{T/2}\left(t - \tfrac{3}{4}T\right)$$

$$S(f) = A \frac{\sin(\pi f T/2)}{\pi f} \cdot e^{-j 2\pi f T/4} - A \frac{\sin(\pi f T/2)}{\pi f} \cdot e^{-j 2\pi f \frac{3}{4}T}$$

$$= A \frac{\sin(\pi f T/2)}{\pi f} \left[e^{-j 2\pi f T/4} - e^{-j 2\pi f \frac{3}{4}T} \right]$$

$$A \frac{T}{2} \operatorname{sinc}\left(\pi f \frac{T}{2}\right)$$

$$X(f) = S(f) * \left[\tfrac{1}{2} \delta(f - f_0) + \tfrac{1}{2} \delta(f + f_0) \right]$$

$$= \frac{A}{2} \frac{\sin\left[\pi (f - f_0) T/2\right]}{\pi (f - f_0)} \left[e^{-j\pi (f - f_0) T/2} - e^{-j\pi (f - f_0)\frac{3}{2}T} \right]$$

$$+ \frac{A}{2} \frac{\sin\left[\pi (f + f_0) T/2\right]}{\pi (f + f_0)} \left[e^{-j\pi (f + f_0) T/2} - e^{-j\pi (f + f_0)\frac{3}{2}T} \right]$$