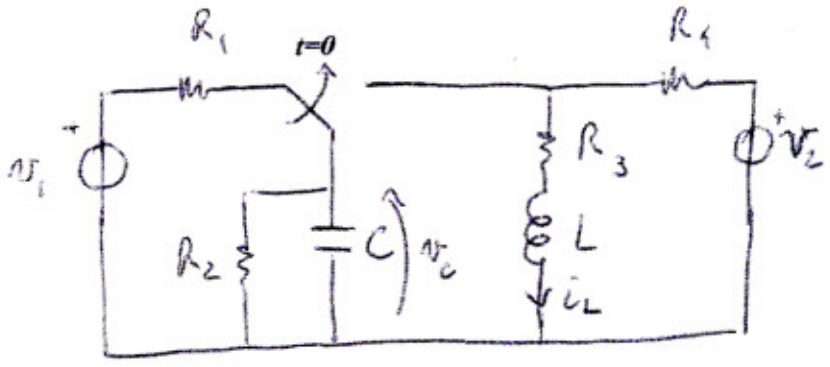
COGNOME (IN STAMPATELLO)	NOME	MATRICOLA	ELT I 15/01/2008	TEMPO 1h10' *3 esercizi a scelta Es. 1 obligatorio*	

Per ogni esercizio riportare sia il risultato che il procedimento utilizzato (utilizzare solamente questo foglio, che va riconsegnato al termine della prova).

1) Dato il circuito in figura

Calcolare e disegnare il grafico di $v_C(t)$ e $i_L(t)$ per $-\infty<t<+\infty$

Commentare i transitori



$L=5H \quad C=2F$

$R_3 = R_4 = 2\ \Omega$

$R_1 = 1\ \Omega$

$R_2 = 9\ \Omega$

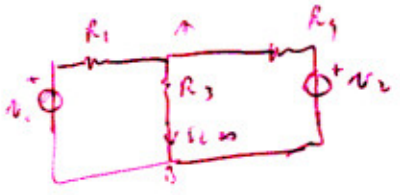
$v_1 = 20\ V$

$v_2 = 10\ V$

$t<0$ $\quad v_C(t) = \frac{R_2}{R_1+R_2} v_1 = \frac{9}{10} \cdot 20 = 18\ V$ $\quad i_L = \frac{v_2}{R_3+R_4} = \frac{10}{4} = 2.5\ A$

$t=0^+$ $\quad v_C(0^+) = v_C(0^-) = 18\ V$ $\quad i_L(0^+) = i_L(0^-) = 2.5\ A$

$t\to\infty$ $\quad v_C(\infty) = 0\ V$

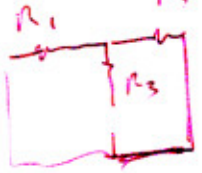


$i_{L\infty} = \frac{v_{AB}}{R_3} = 6.25\ A$

$v_{AB} = \frac{\frac{v_1}{R_1} + \frac{v_2}{R_4}}{\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_4}} = \frac{20+5}{1+\frac{1}{2}+\frac{1}{2}} = 12.5\ V$

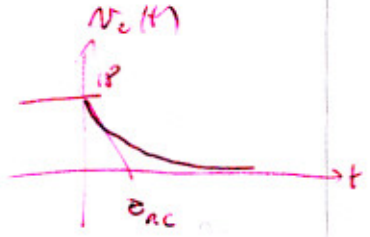
$\tau_{RC} = R_2 C = 9 \cdot 2 = 18\ sec.$

$\tau_{RL} = L/R_{eq} = \frac{5}{8} \cdot 3 = \frac{15}{8} = 1.9\ sec$



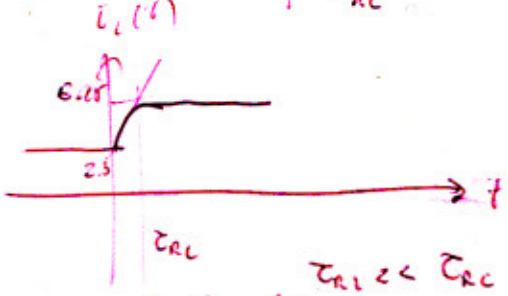
$R_{eq} = R_3 + R_1 \| R_4 = 2 + \frac{1 \cdot 2}{3} = \frac{8}{3}\ \Omega$

$v_C(t) = (v_C(0^+) - v_{C\infty})\, e^{-t/\tau_{RC}} + v_{C\infty} = 18\, e^{-\frac{1}{18}t}$



$i_L(t) = (2.5 - 6.25)\, e^{-\frac{8}{15}t} + 6.25$

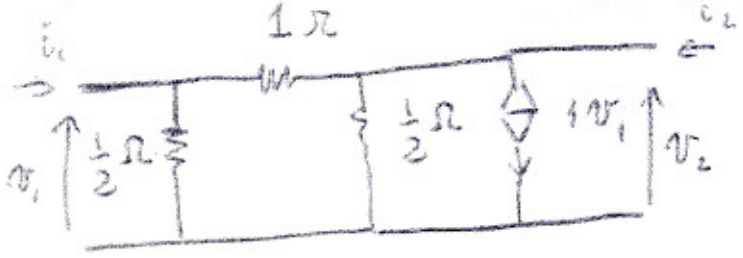
$= -3.75\, e^{-\frac{8}{15}t} + 6.25$



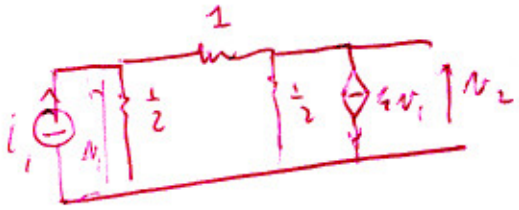
$\tau_{RL} < \tau_{RC}$

più veloce il transitorio RL

2) Calcolare la matrice di resistenze del doppio bipolo inerte



$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$



$i_2 = 0$

spegn. (i_1) $\quad v_1' = i_1 \cdot \frac{1}{2} /\!/ \frac{3}{2} = \frac{3}{8} i_1$ ($\frac{3}{2} \to \frac{3}{8}$)

$$v_2' = \frac{\frac{1}{2}}{\frac{3}{2}} \cdot v_1' = \frac{v_1'}{3} = \frac{1}{8} i_1$$

($4v_1$) $\quad v_2'' = -4v_1 \left(\frac{1}{2} /\!/ \frac{3}{2} \right) = -\frac{3}{2} v_1$ ($\frac{3}{2} \to \frac{3}{8}$)

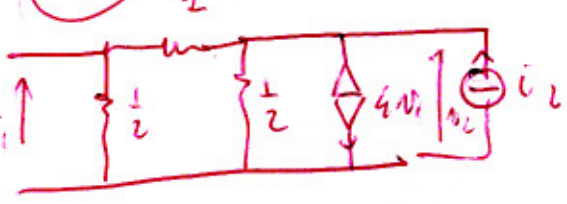
$$v_1'' = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{2}} v_2'' = \frac{v_2''}{3} = -\frac{1}{2} v_1$$

$v_1 = v_1' + v_1'' = \frac{3}{8} i_1 - \frac{1}{2} v_1 \Rightarrow v_1 = \frac{1}{4} i_1 \Rightarrow R_{11} = \left. \frac{v_1}{i_1} \right|_{i_2=0} = \frac{1}{4}\Omega$

$v_2 = v_2' + v_2'' = \frac{1}{8} i_1 - \frac{3}{2} v_1 \Rightarrow v_2 = \frac{1}{8} i_1 - \frac{3}{8} i_1 = -\frac{1}{4} i_1$

$$R_{21} = \left. \frac{v_2}{i_1} \right|_{i_2=0} = -\frac{1}{4}\Omega$$

$i_1 = 0$

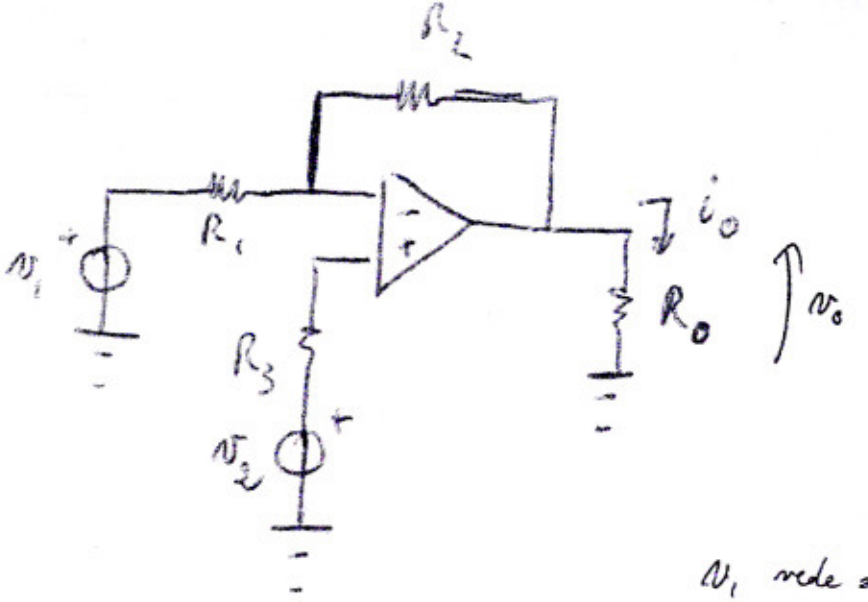


$v_1 = (i_2 - 4v_1) \cdot \left(\frac{1}{2} /\!/ \frac{3}{2}\right) \cdot \frac{\frac{1}{2}}{\frac{1}{2}+1} = (i_2 - 4v_1) \cdot \frac{3}{8} \cdot \frac{1}{2}$ ($\frac{3}{2} \to \frac{3}{8}$; v_2; partitore)

$v_1 = \frac{1}{12} i_2 \Rightarrow R_{12} = \left. \frac{v_1}{i_2} \right|_{i_1=0} = \frac{1}{12}\Omega$

$v_2 = v_1 \frac{1}{3} \Rightarrow v_2 = \frac{1}{4} i_2 \Rightarrow R_{22} = \left. \frac{v_2}{i_2} \right|_{i_1=0} = \frac{1}{4}\Omega$

3) Dato il circuito in figura
Calcolare I_o



V_1 vede stadio inv.

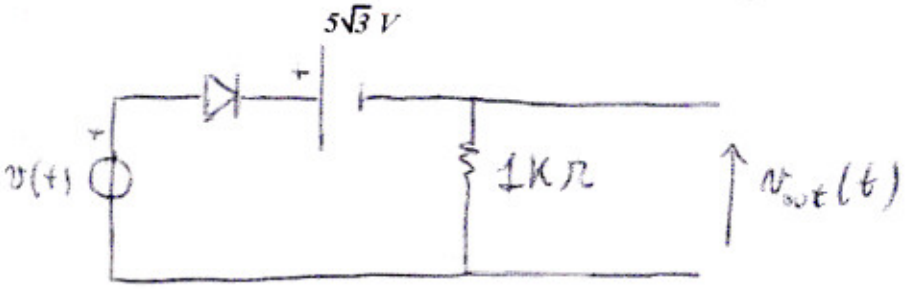
V_2 " " non inv.

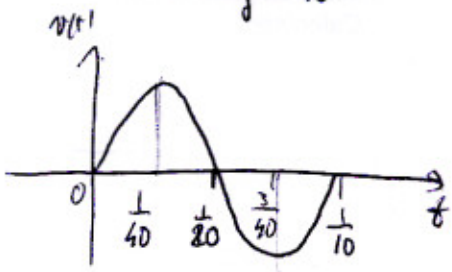
$$V_o = -V_1 \frac{R_2}{R_1} + V_2\left(1 + \frac{R_2}{R_1}\right) =$$

$$i_o = \frac{V_o}{R_o} = \frac{-V_1 \frac{R_2}{R_1} + V_2\left(1 + \frac{R_2}{R_1}\right)}{R_o}$$

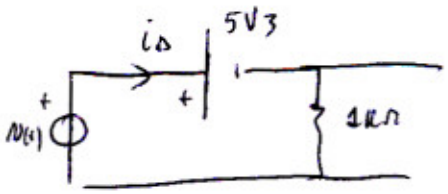
4) Calcolare l'andamento $V_{out}(t)$ in funzione del tempo e disegnarne un grafico quotato. [v(t)=10 sin(ω t) V; f=10Hz]

$T = \frac{1}{f} = \frac{1}{10}$ sec



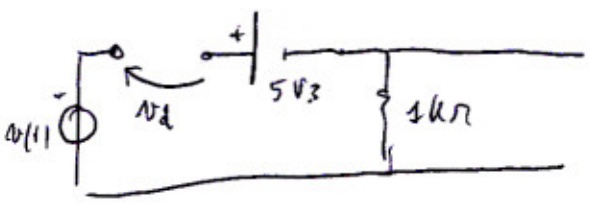


ON



$V_{out}(t) = v(t) - 5\sqrt{3}$

OFF



$V_{out}(t) = 0$

ON $\quad i_D(t) > 0 \Rightarrow v(t) - 5\sqrt{3} > 0$

$10 \sin \omega t - 5\sqrt{3} > 0$

$\alpha = \omega t$

$\sin \omega t > \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

$\frac{\pi}{3} \leq \alpha \leq \frac{2\pi}{3}$

$\omega t > \frac{\pi}{3}$ $\qquad 2\pi f t > \frac{\pi}{3}$

$t > \frac{1}{6 \cdot 10} = \frac{1}{60}$ sec

OFF $\quad V_d < 0$ $\qquad V_d(t) = v(t) - 5\sqrt{3} < 0$ $\qquad$ Vout(t)=0

Quindi