ESERCIZI PROPOSTI

1. *Calcolare i valori delle seguenti espressioni:*

- $5\sin\frac{\pi}{2}+\frac{2}{3}\text{tg}\frac{\pi}{4}-\frac{1}{4}\cos\frac{\pi}{3}+5\cos(\pi)$ $\left[\mathbf{R}.\ \frac{13}{24}\right]$
- $\sqrt{3}\sin\frac{\pi}{3}-\cos\frac{5\pi}{6}+\frac{1}{\sqrt{2}}\cos\left(-\frac{\pi}{4}\right)+2\,\text{tg}\frac{\pi}{3}$ $\left[\mathbf{R}.\ \frac{4+5\sqrt{3}}{2}\right]$
- $\cos\frac{2\pi}{3}-\sin\left(-\frac{5\pi}{6}\right)-\text{tg}\frac{\pi}{4}+\text{tg}-(3\pi)+\text{tg}\left(-\frac{7\pi}{6}\right)$ $\left[\mathbf{R}.\ -\frac{3+\sqrt{3}}{3}\right]$

2. *Verificare le seguenti identità*

- $\cos\left(\frac{\pi}{2}-x\right)+\cos(\pi-x)+\sin(\pi+x)=-3\cos x+2\cos(-x)$
- $\sin\left(\frac{\pi}{2}+x\right)\cos(\pi-x)=\sin\left(\frac{\pi}{2}+x\right)\cos(\pi+x)$
- $\cos(2x)=1-2\sin^2 x$
- $\cos(2x)=2\cos^2 x-1$
- $\text{tg}\left(\frac{\pi}{4}-x\right)\cdot\text{tg}\left(\frac{\pi}{4}+x\right)=1$

- $\operatorname{tg}^2 x = \dfrac{1-\cos(2x)}{1+\cos(2x)}$
- $\cos^2 x = \dfrac{1}{1+\operatorname{tg}^2 x}$
- $\sin^2 x = \dfrac{\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x}$
- $\dfrac{\cos(2x)}{1+\sin(2x)} = \dfrac{\operatorname{cotg} x - 1}{\operatorname{cotg} x + 1}$
- $\cos\left(\dfrac{\pi}{3} - x\right) + \cos\left(\dfrac{\pi}{3} + x\right) = \cos x$

3. Risolvere in **R** *le seguenti equazioni:*

- $2\cos x = -1$ $\left[\mathbf{R}.\ x = \pm\dfrac{2}{3}\pi + 2k\pi\right]$
- $2\sin(2x) = \sqrt{3}$ $\left[\mathbf{R}.\ x = \dfrac{\pi}{3} + k\pi \cup x = \dfrac{\pi}{6} + k\pi\right]$
- $2\cos^2 x = 1$ $\left[\mathbf{R}.\ x = \pm\dfrac{\pi}{4} + 2k\pi \cup x = \pm\dfrac{3\pi}{4} + 2k\pi\right]$

- $\sin\left(\frac{p}{4} - x\right) = \frac{\sqrt{3}}{2}$ $\left[\mathbf{R}.\ x = -\frac{p}{12} + 2kp \cup x = -\frac{5p}{12} + 2kp\right]$
- $\cos\left(x + \frac{p}{6}\right) = \cos\left(2x - \frac{p}{4}\right)$ $\left[\mathbf{R}.\ x = \frac{5p}{12} + 2kp \cup x = -\frac{p}{36} + k\frac{2p}{3}\right]$
- $tg\left(\frac{p}{6} - 2x\right) = \mathrm{tg}\left(3x - \frac{p}{3}\right)$ $\left[\mathbf{R}.\ x = \frac{p}{10} + k\frac{p}{5}\right]$
- $6\sin^2 x - 13\sin x + 5 = 0$ $\left[\mathbf{R}.\ x = \frac{p}{6} + 2kp \cup x = \frac{5p}{6} + 2kp\right]$
- $2\cos^2(2x) + \cos(2x) = 0$ $\left[\mathbf{R}.\ x = \frac{p}{4} + k\frac{p}{2} \cup x = \pm\frac{p}{3} + kp\right]$
- $\sin x + \cos x - 1 = 0$ $\left[\mathbf{R}.\ x = \frac{p}{2} + 2kp \cup x = 2kp\right]$
- $\sqrt{3}\sin x = \cos x$ $\left[\mathbf{R}.\ x = \frac{p}{6} + kp\right]$
- $4\sin^2 x - 2\sqrt{3}\sin x\cos x - 2\cos^2 x - 1 = 0$ $\left[\mathbf{R}.\ x = -\frac{p}{6} + kp \cup x = \frac{p}{3} + kp\right]$
- $2\sin^2 x - \cos x = 1$ $\left[\mathbf{R}.\ x = \pm\frac{p}{3} + 2kp \cup x = p + 2kp\right]$

4. *Risolvere le seguenti disequazioni negli intervalli a fianco indicati:*

- $2\sin x < \sqrt{3}$ $\qquad [0,2\pi]$ $\qquad \left[\mathbf{R}.\left[0,\frac{\pi}{3}\right)\cup\left(\frac{2\pi}{3},2\pi\right]\right]$
- $\cos x > \pi$ $\qquad [0,2\pi]$ $\qquad [\mathbf{R}.\ \varnothing]$
- $\cos x \geq -\frac{1}{2}$ $\qquad [-\pi,\pi]$ $\qquad \left[\mathbf{R}.\left[-\frac{2\pi}{3},\frac{2\pi}{3}\right]\right]$
- $\mathrm{tg}x < -1$ $\qquad [0,\pi]$ $\qquad \left[\mathbf{R}.\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\right]$
- $3\mathrm{tg}^2 x < 1$ $\qquad \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ $\qquad \left[\mathbf{R}.\left(-\frac{\pi}{6},\frac{\pi}{6}\right)\right]$
- $\cos^2 x \leq \frac{3}{4}$ $\qquad [0,2\pi]$ $\qquad \left[\mathbf{R}.\left[\frac{\pi}{6},\frac{5\pi}{6}\right]\cup\left[\frac{7\pi}{6},\frac{11\pi}{6}\right]\right]$
- $2\cos^2 x - 5\cos x - 3 > 0$ $\qquad [0,2\pi]$ $\qquad \left[\mathbf{R}.\ \left(\frac{2\pi}{3},\frac{4\pi}{3}\right)\right]$
- $2\sin x\cos x - \cos x \geq 0$ $\qquad [0,2\pi]$ $\qquad \left[\mathbf{R}.\left[\frac{\pi}{6},\frac{\pi}{2}\right]\cup\left[\frac{5\pi}{6},\frac{3\pi}{2}\right]\right]$
- $\frac{1-\cos x}{\sqrt{3}\mathrm{tg}x - 1} < 0$ $\qquad [-\pi,\pi]$ $\qquad \left[\mathbf{R}.\left[-\pi,-\frac{5\pi}{6}\right)\cup\left(-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{6}\right)\cup\left(\frac{\pi}{2},\pi\right]\right]$